# **Radicals/Roots Cheat Sheet** (Dan's Test Prep)

Must-Know Basics:

A radical (or root) is basically the "opposite" operation of an exponent.
Much like division undoes multiplication, a root undoes an exponent.

Examples: $\sqrt{x^2} = x$ $\sqrt[3]{x^3} = x$

$\sqrt[4]{2^4} = 2$ $\sqrt[5]{-32} = \sqrt[5]{(-2)^5} = -2$

This fact becomes clearer when you realize that roots/radicals can be rewritten as fractional exponents (by simply shifting the number on the root to the denominator of the exponent).

Examples: $\sqrt{x} = x^{\frac{1}{2}}$ $\sqrt[3]{x^6} = x^{\frac{6}{3}} = x^2$

$\sqrt[3]{8^2} = 8^{\frac{2}{3}} = 2^{3 \cdot \frac{2}{3}} = 2^2 = 4$

## Rationalizing an expression:

A radical in a denominator is technically incorrect, so you'll need to multiply both the top and bottom of an expression by that radical to "rationalize" or get rid of it. The answer choices will likely be rationalized.

*If the denominator has addition/subtraction, you'll multiply by the conjugate (same thing but flip the sign)*

Examples: $\frac{x}{\sqrt{7}} = \frac{x}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{x\sqrt{7}}{7}$ $\frac{5}{3\sqrt{2}} = \frac{5}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{3 \cdot 2} = \frac{5\sqrt{2}}{6}$

$$\frac{10}{3-\sqrt{2}} = \frac{10}{3-\sqrt{2}} \cdot \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{30+10\sqrt{2}}{9-2} = \frac{30+10\sqrt{2}}{7} = \frac{30}{7} + \frac{10\sqrt{2}}{7}$$

Must-Know Rules:

| Radical Rule | Algebraic Example | Numeric Example |
|---|---|---|
| When multiplying something under a radical, you can split apart the terms, and vice-versa | $\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$<br>$\sqrt{y} \cdot \sqrt{z} = \sqrt{yz}$ | $\sqrt{75} = \sqrt{25} \cdot \sqrt{3} = 5\sqrt{3}$<br>$\sqrt{6} \cdot \sqrt{24} = \sqrt{6 \cdot 24} = \sqrt{144}$ |
| When a fraction is under a radical, it can be split into two separate radicals | $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$ | $\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$ |
| Radicals can be written as fractional exponents and vice versa | $\sqrt{x} = x^{\frac{1}{2}}$ | $\sqrt{4} = 4^{\frac{1}{2}} = 2^{2 \cdot \frac{1}{2}} = 2$ |
| The numerator of a fractional exponent corresponds to the power to which the base is raised; the denominator corresponds to the root | $\sqrt[z]{x^y} = x^{\frac{y}{z}}$<br>$x^{\frac{a}{b}} = \sqrt[b]{x^a}$ | $\sqrt[3]{(2x)^5} = (2x)^{\frac{5}{3}}$<br>$5^{\frac{4}{5}} = \sqrt[5]{5^4}$ |
| When taking the square root of a single number, the answer is always positive, but in an equation, it can be + or - | $x^2 = y \rightarrow \sqrt{x^2} = \pm\sqrt{y}$<br>$\rightarrow \quad x \quad = \pm\sqrt{y}$ | $x^2 = 9 \ \rightarrow \ x = \pm 3$<br>but $\sqrt{9} = +3$ only |
| To rationalize an expression containing a radical in the denominator, multiply both the top and bottom by that radical | $\frac{x}{\sqrt{y}} = \frac{x}{\sqrt{y}} \cdot \frac{\sqrt{y}}{\sqrt{y}} = \frac{x\sqrt{y}}{y}$ | $\frac{5}{\sqrt{2}} = \frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$ |