Circles Cheat Sheet (Dan's Test Prep)

The circle questions you see on the SAT will typically deal with one of two categories: circle theorems and circle equations.

- Circle Equations: Deal with circles on a coordinate plane (center, distance, radius)
- Circle Theorems: Deal with arc lengths, area, central angles, and sectors

Must-Know Circle Terms:

Radius: The distance from the center of a circle to a point on its edge.

Diameter: The distance from one point on the circle, through its center, to a point on the opposite end. The diameter is always twice the radius.

Circumference: The distance around the circle (essentially the perimeter).

Area: The extent of the space that the circle encompasses in the 2D plane.

Basic Circle Equations:

Radius of a circle:	r
Diameter of a circle:	$D = 2r$
Circumference of a circle:	$C = 2\pi r = \pi D$
Area of a circle:	$A = \pi r^2$

Equation of a Circle in the (x, y)-Plane:

$(x - h)^2 + (y - k)^2 = r^2$ Where (h, k) is the center of the circle and r is the radius of the circle *(Standard Form)*

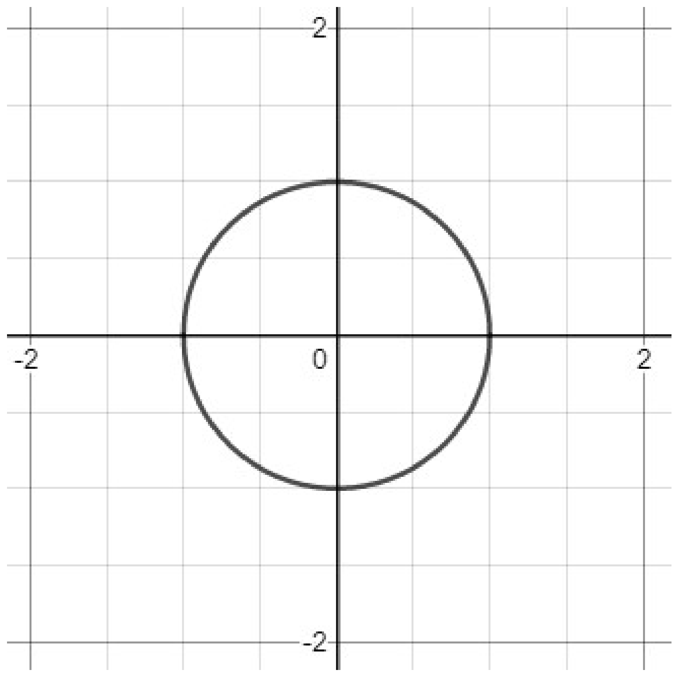
Make sure you are very familiar with this equation and how it is used. The SAT can test you on this in a tricky way, such as giving you the center of the circle and an endpoint, requiring you to solve for the radius to figure out the equation.

Example Circle Graphs (Helpful Visualizations):

Here are some examples of what changing your circle equation can do to its graph.

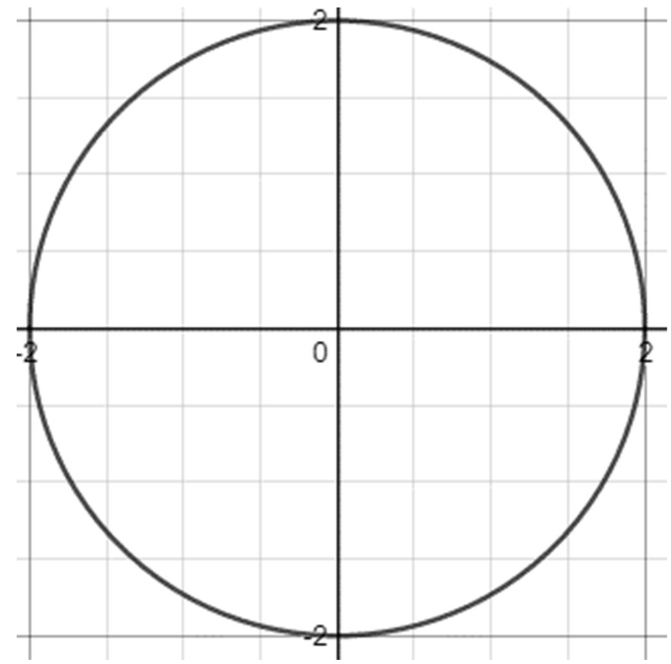
$$x^2 + y^2 = 1$$

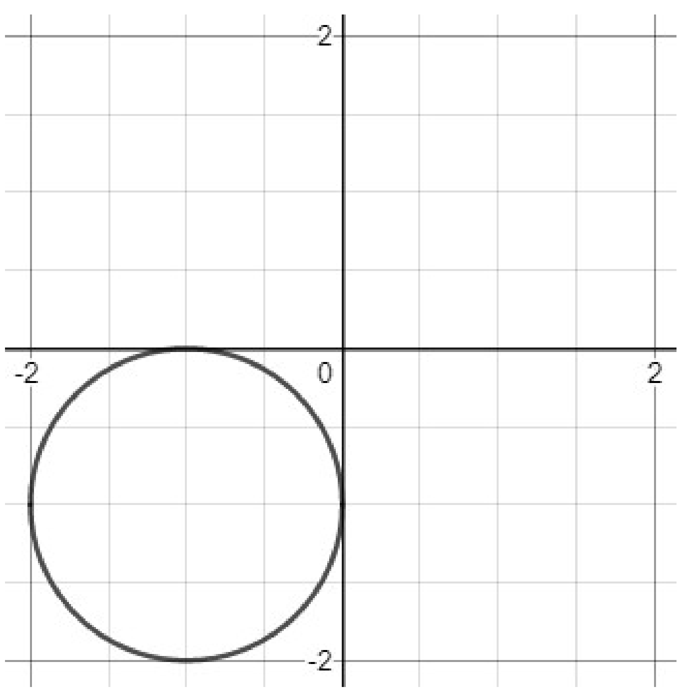
$$h = 0,\ k = 0,\ r = 1$$



$$x^2 + y^2 = 4$$

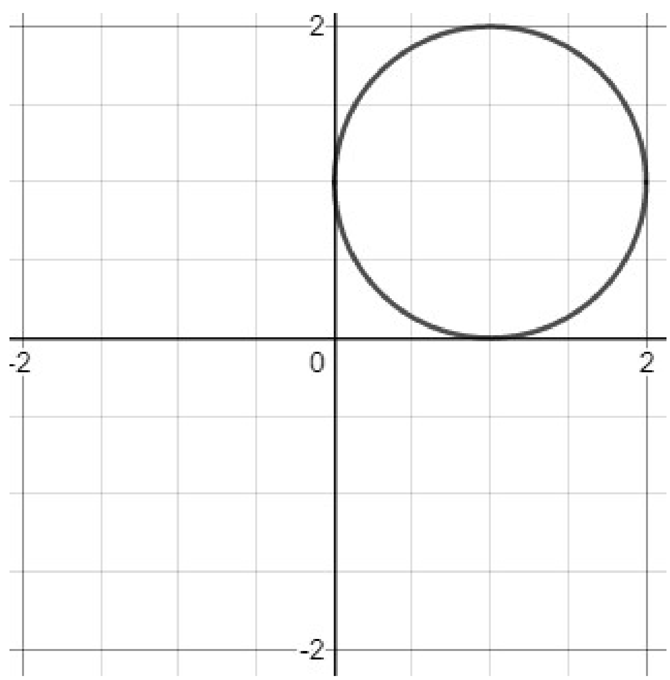
$$h = 0,\ k = 0,\ r = 2$$





$$h = -1,\ k = -1,\ r = 1$$

$$(x + 1)^2 + (y + 1)^2 = 1$$



$$h = 1,\ k = 1,\ r = 1$$

$$(x - 1)^2 + (y - 1)^2 = 1$$

General Form and Completing the Square:

Sometimes, you'll see an equation of a circle in the following form:

$$x^2 + y^2 + Ax + By + C = 0 \qquad \textit{(General Form)}$$

This equation still graphs a circle, just like our previous one, but now it is a little less clear what the center and radius are. To make our lives easier, we can "complete the square" to put this equation into the Standard Form.

I'll illustrate with an example to make things a little less abstract.
Suppose our General Form equation is $x^2 + y^2 + 2x - 6y + 6 = 0$

I'll start by moving the constant over to the right side.

$x^2 + y^2 + 2x - 6y = -6$

Next, I'll group the x and y terms together.

$x^2 + 2x + y^2 - 6y = -6$

Now for the process of completing the square. To complete the square, we take our coefficient on the x or y term, we cut in half, and square it. Then, add these new constants to both sides of the equation to maintain the balance.

Since our coefficient on the is x term is 2, we divide by 2 ($\frac{2}{2} = 1$) and square it to get 1. Repeat this process to complete the square for y. Our y term is $6y$. The coefficient is therefore 6. 6 divided by 2 is 3, and 3 squared is 9.

$x^2 + 2x + 1 + y^2 - 6y + 9 = -6 + 1 + 9$

Now you may start to notice something interesting. After completing the square for both x and y, we can actually factor our terms into General Form by recognizing our newly formed perfect squares. *Now we see that h = -1, k = 3, r = 2*

$(x + 1)^2 + (y - 3)^2 = 4$ or $(x + 1)^2 + (y - 3)^2 = 2^2$

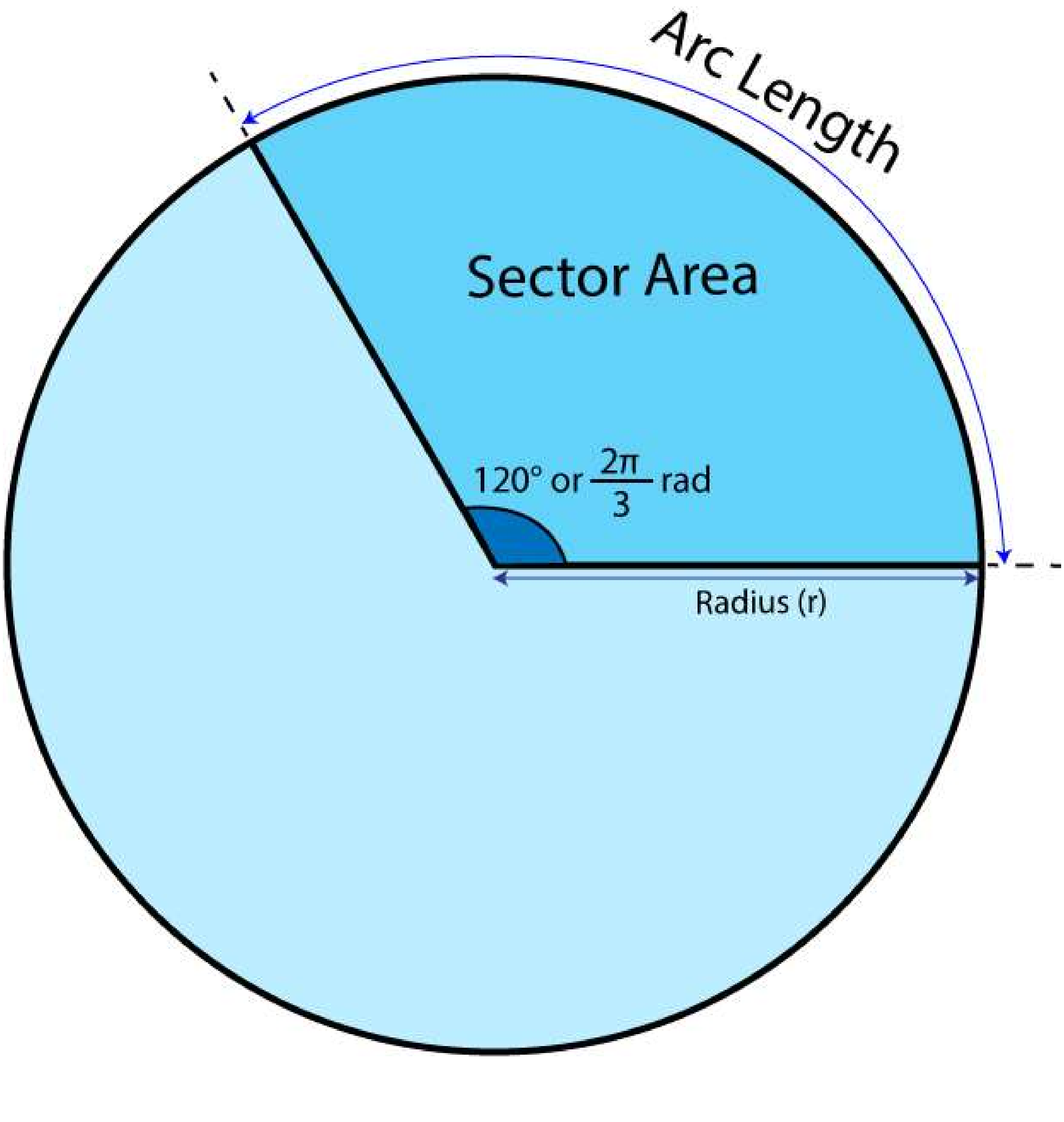
Circle Theorems and Must-Know Equations:

Let's start by reviewing some more terms you'll need to know.

<u>Arc length</u>: The distance between two points along the curve of a circle

<u>Sector</u>: A portion of area captured by an angle traveling outward from the center of a circle to its edge

This can be really confusing, so I created a visualization to help clarify it:



Here are some formulas you absolutely need to know in order to solve these questions properly. The SAT does not provide these for you!

Angle Measure:	Radians	Degrees
Arc Length (s)	$s = r\theta$	$s = \frac{\theta}{360°} \cdot 2\pi r$
Area of a Sector (A)	$A = \frac{1}{2} r^2 \theta$	$A = \frac{\theta}{360°} \cdot \pi r^2$

Remember that *s* is the arc length, *A* is the area of a sector, and θ is the angle coming outwards from the center of the circle.

You should also know that you can easily convert between radians and degrees using this simple trick:

If you're going from radians to degrees, multiply the value by $\left(\frac{180°}{\pi}\right)$

If you're going from degrees to radians, multiply the value by $\left(\frac{\pi}{180°}\right)$

Notice how the original units will cancel out, leaving you with your desired result.

I prefer to use the formulas in radians, especially if I encounter a question with an angle in radians, but I think the formulas in degrees are much more intuitive.

I always think it helps to obtain an intuitive understanding of the math behind the concepts. You too can obtain this type of understanding!

Notice that arc length is simply the circumference of the circle $(2\pi r)$ times the proportion of the circle covered by your angle $\left(\frac{\theta}{360°}\right)$. Similarly, the area of a sector is simply the area of the circle (πr^2) times the proportion of the circle covered by your angle $\left(\frac{\theta}{360°}\right)$. Hopefully now the equations are a little clearer.